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Supercurrent Interactions in Noncommutative Yang-Mills and IIB Matrix Model

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Abstract

It is known that noncommutative Yang-Mills is equivalent to IIB matrix model with a noncommutative background, which is interpreted as a twisted reduced model. In noncommutative Yang-Mills, long range interactions can be seen in nonplanar diagrams after integrating high momentum modes. These interactions can be understood as block-block interactions in the matrix model. Using this relation, we consider long range interactions in noncommutative Yang-Mills associated with fermionic backgrounds. Exchanges of gravitinos, which couple to a supersymmetry current, are examined.

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1 Introduction

Several kinds of Matrix Model have been proposed[1, 2] to study the nonperturbative aspects of string theory or M theory. These proposals are based on the developments of D-brane physics. D-branes have been shown to play a fundamental role in nonperturbative string theory[3, 4]. A notable point is that supersymmetric gauge theory can be obtained on their world-volume as their low energy effective theory. The idea of matrix models is that supersymmetric gauge theory can describe string or M theory.

IIB Matrix Model is one of these proposals[2]. It is a large N reduced model[5] of ten-dimensional supersymmetric Yang-Mills theory and the action has a matrix regularized form of the Green-Schwarz action of IIB superstring. It is postulated that it gives the constructive definition of type IIB superstring theory. This model has ten dimensional $\mathcal{N} = 2$ supersymmetry, which implies the existence of gravitons. In the matrix model, gravitational interactions arise as quantum effects. In fact, the leading long range interaction in the matrix model is identified with the supergraviry results. Gravitons couple to energy-momentum tensor and the interaction between the separate objects exhibits the graviton exchange process[2]. We expect that IIB matrix model should reproduce the interactions which are mediated by the whole multiplet in IIB supergravity. In [2], one-loop effective action is calculated only for bosonic backgrounds. By considering fermionic backgrounds[20, 21], fermionic particles such as gravitinos or dilatinos are expected to be seen in the matrix model calculation. Hence it is important to consider fermionic backgrounds to check that the IIB matrix model can reproduce the interactions expected in IIB supergravity.

Recently, noncommutative Yang-Mills theories have been studied in many situations. It first appeared within the framework of toroidal compactification of Matrix theory[7]. It is discussed in [8] that the world volume theory on D-branes with NS-NS two-form background is described by noncommutative Yang-Mills theory. In matrix models, space-time coordinates are represented by matrices. Therefore the noncommutativity appears naturally and matrix models are considered to be closely related to noncommutative geometry. It was shown[12, 17, 18] that in the matrix model picture noncommutative Yang-Mills theory is equivalent to twisted reduced models[6]. Twisted reduced models are obtained by expanding the model around noncommutative backgrounds. A noncommutative background is a D-brane-like background which is a solution of equation of motion and preserves a part of supersymmetry. It is well known that gauge theory is realized in the world-volume of D-branes as their low energy effective theory. In IIB matrix model, gauge theory is realized as twisted reduced models. Noncommutative Yang-Mills as a twisted reduced model has been studied in [13, 14, 15, 16, 19].

Noncommutative field theory has a lot of interesting properties which are absent in ordinary field theory. While amplitudes for planar diagrams in the noncommutative theory

are the same as those in the commutative theory up to a phase factor associated with external lines, amplitudes for nonplanar diagrams in the noncommutative theory are ultraviolet finite due to the oscillation of the phase factor[6, 9, 10, 13]. Perturbative dynamics of noncommutative field theory has been further studied in [24, 25] and it is pointed out that the effective action has infrared singular behavior in nonplanar diagrams. After integrating high momentum modes, long range interactions, which are absent in ordinary field theories, can be obtained. This infrared singular behavior may be related with the propagation of massless particles in the bulk. This behavior reminds us of the channel duality in string theory. High momentum modes at open string one loop level on the brane corresponds to the exchange of low momentum modes in closed string, which propagates in the bulk. A nonplanar one-loop diagram is topologically equivalent to a tree level diagram in closed string theory. This interaction can be understood as block-block interactions in the matrix model picture[14]. These long range interactions, or the propagation of massless particles, are universal property of noncommutative field theories and the matrix model.

In this paper, we consider supercurrent interactions in noncommutative Yang-Mills and IIB matrix model using the formulation of noncommutative Yang-Mills as twisted reduced models. We find that block-block interactions at order $1/r^8$ in the matrix model with fermionic background give gravitino exchange processes at order $1/r^9$ in noncommutative Yang-Mills. The interaction which decays as $1/r^9$ is interpreted as the propagation of a massless fermion in ten dimensions and does not depend on the information of the extension of the matrix eigenvalues. Then it is presented that one of the gravitinos in IIB supergravity couples to a supersymmetry current which is a Noether current associated with supersymmetry in IIB matrix model. The organization of this paper is as follows. In section 2, we review IIB matrix model and its relation to noncommutative Yang-Mills theory. In the matrix model picture, noncommutative Yang-Mills is equivalent to twisted reduced model. In section 3, we consider the long range interactions with fermionic backgrounds in IIB matrix model and noncommutative Yang-Mills. These long range interactions arise from nonplanar diagrams in noncommutative Yang-Mills. Long range interactions in noncommutative Yang-Mills which decay as $1/r^9$ are obtained. It is shown that this interactions are due to the gravitino exchange and this gravitino couples to a supersymmetry current in section 4. The interactions between supersymmetry currents via a gravitino exchange in ten-dimensional supergravity are computed. Then we compare the matrix model calculation with a supergravity calculation. Section 5 is devoted to conclusions and discussions.

2 IIB matrix model and noncommutative Yang-Mills

In this section, we review IIB matrix model[2, 11] and its relation to noncommutative Yang-Mills[12, 13, 14].

We begin with the action which is defined by the following form:

$$S=-\frac{1}{g^2}Tr\left(\frac{1}{4}\left[A_\mu,A_\nu\right]\left[A^\mu,A^\nu\right]+\frac{1}{2}\bar{\psi}\Gamma^\mu\left[A_\mu,\psi\right]\right). \tag{1}$$

Here ψ is a ten dimensional Majorana-Weyl spinor field, and A_μ and ψ are $N\times N$ hermitian matrices. This model is the large N reduced model of ten-dimensional $\mathcal{N}$=1 $U(N)$ supersymmetric Yang-Mills theory. This is based on the observation that in the large N t'Hooft limit $U(N)$ gauge theory is equivalent to its reduced model which is obtained by reducing the space-time volume to a single point[5].

This model has the manifest ten dimensional Lorentz symmetry and following symmetries:

$$\begin{aligned}\delta^{(1)}\psi&=\frac{i}{2}\left[A_\mu,A_\nu\right]\Gamma^{\mu\nu}\epsilon,\\ \delta^{(1)}A_\mu&=i\bar{\epsilon}\Gamma^\mu\psi,\end{aligned} \tag{2}$$

and

$$\begin{aligned}\delta^{(2)}\psi&=\xi,\\ \delta^{(2)}A_\mu&=0,\end{aligned} \tag{3}$$

and translation symmetry:

$$\delta A_\mu=c_\mu\mathbf{1}. \tag{4}$$

If we interpret the eigenvalues of A_μ as the space-time coordinates, we can regard the above symmetry as $\mathcal{N}=2$ supersymmetry[2]. We take a linear combination of $\delta^{(1)}$ and $\delta^{(2)}$ as

$$\begin{aligned}\tilde{\delta}^{(1)}&=\delta^{(1)}+\delta^{(2)},\\ \tilde{\delta}^{(2)}&=i\left(\delta^{(1)}-\delta^{(2)}\right).\end{aligned} \tag{5}$$

We can obtain $\mathcal{N}=2$ supersymmetry algebra:

$$\begin{aligned}\left(\tilde{\delta}^{(i)}_\epsilon\tilde{\delta}^{(j)}_\xi-\tilde{\delta}^{(j)}_\xi\tilde{\delta}^{(i)}_\epsilon\right)\psi&=0,\\ \left(\tilde{\delta}^{(i)}_\epsilon\tilde{\delta}^{(j)}_\xi-\tilde{\delta}^{(j)}_\xi\tilde{\delta}^{(i)}_\epsilon\right)A_\mu&=2i\bar{\epsilon}\Gamma^\mu\xi\delta_{ij}.\end{aligned} \tag{6}$$

The classical equations of motion of (1) are

$$\Gamma_\mu[A_\mu,\psi]=0, \tag{7}$$

$$[A_\mu, [A_\mu, A_\nu]] = -\bar{\psi}\Gamma_\nu\psi. \tag{8}$$

In the latter part of this section, we briefly review the formulation[12] of noncommutative gauge theory as twisted reduced models. We expand the theory around the following classical solution,

$$[\hat{p}^\mu, \hat{p}^\nu] = iB^{\mu\nu}, \tag{9}$$

where $B_{\mu\nu}$ is anti-symmetric tensor and proportional to a unit matrix. This is a solution of (8) with $\psi = 0$ and corresponds to a BPS background ($\xi = \pm 1/2B^{\mu\nu}\epsilon$) [2]. We assume the rank of $B_{\mu\nu}$ to be d and define its inverse $C^{\mu\nu}$ in d dimensional subspace. $\hat{p}^\mu$ satisfy the canonical commutation relations and span the d dimensional phase space. The volume of the phase space is $V_p = n(2\pi)^{d/2}\sqrt{\det B}$. Then we expand $A_\mu = \hat{p}_\mu + \hat{a}_\mu$ and Fourier-decompose $\hat{a}_\mu$ and $\hat{\psi}$ as

$$\hat{a}_\mu = \sum_k \tilde{a}_\mu(k) \exp(iC^{\mu\nu}k_\mu\hat{p}^\nu), \tag{10}$$

$$\hat{\psi} = \sum_k \tilde{\psi}(k) \exp(iC^{\mu\nu}k_\mu\hat{p}^\nu). \tag{11}$$

$\exp(iC^{\mu\nu}k_\mu\hat{p}^\nu)$ is the eigenstate of $P_\mu = [\hat{p}^\mu, \cdot]$ with eigenvalue k_μ. The Hermiticity requires that $\tilde{a}^*_\mu(k) = \tilde{a}_\mu(-k)$ and $\tilde{\psi}^*_\mu(k) = \tilde{\psi}_\mu(-k)$. Let Λ be the extension of each $\hat{p}_\mu$. The volume of one quantum in this phase space is $\Lambda^d/N = \lambda^d$ where λ is the spacing of the quanta, say, noncommutative scale. B , which is the component of $B_{\mu\nu}$, is related to λ as $B = \lambda^2/2\pi$. k^μ is quantized in the unit of $k^{min}_\mu = \Lambda/N^{2/d} = \lambda/N^{1/d}$. The range of k_μ is restricted as $-N^{1/d}\lambda/2 \le k_\mu \le N^{1/d}\lambda/2$.

Consider the map from a matrix to a function as

$$\hat{a}_\mu \to a_\mu(x) = \sum_k \tilde{a}_\mu(k) \exp(ik_\mu x^\mu). \tag{12}$$

$$\hat{\psi} \to \theta(x) = \sum_k \tilde{\theta}(k) \exp(ik_\mu x^\mu). \tag{13}$$

We consider this field as the gauge field in noncommutative gauge theory. Under this map, we obtain the following map,

$$\hat{a}\hat{b} \to a(x) \star b(x), \tag{14}$$

where $\star$ is the star product defined as follows,

$$a(x) \star b(x) \equiv \exp\left(\frac{iC^{\mu\nu}}{2}\frac{\partial^2}{\partial\xi^\mu\partial\eta^\nu}\right) a(x+\xi)b(x+\eta) \mid_{\xi=\eta=0} . \tag{15}$$

Tr over matrices can be mapped on the integration over functions as

$$Tr[\hat{a}] = \sqrt{\det B}\left(\frac{1}{2\pi}\right)^{\frac{d}{2}}\int d^d x a(x). \tag{16}$$

Using these rules, the adjoint operator of $\hat{p}_\mu + \hat{a}_\mu$ is mapped to the covariant derivative:

$$[\hat{p}_\mu + \hat{a}_\mu, \hat{o}] \to \frac{1}{i}\partial_\mu o(x) + a_\mu(x)\star o(x) - o(x)\star a_\mu(x) \equiv \frac{1}{i}\left[D_\mu, o(x)\right]_\star, \tag{17}$$

and

$$f_{\mu\nu} = i[A_\mu, A_\nu] \to -B_{\mu\nu} + \partial_\mu a_\nu - \partial_\nu a_\mu + i[a_\mu, a_\nu]_\star. \tag{18}$$

The equations of motion (7) and (8) of the matrix model are mapped to

$$\Gamma_\mu[D_\mu, \theta]_\star = 0, \tag{19}$$

$$[D_\mu, f_{\mu\nu}]_\star = (\Gamma_\nu)_{\alpha\beta}\bar{\theta}_\alpha \star \theta_\beta. \tag{20}$$

By applying these rules to the action (1), $U(1)$ noncommutative Yang-Mills theory has been obtained:

$$\begin{aligned}
&-\frac{1}{4g^2}Tr\left[A_\mu, A_\nu\right]\left[A^\mu, A^\nu\right] \\
&\to \frac{dNB^2}{4g^2} - \sqrt{\det B}(\frac{1}{2\pi})^{\frac{d}{2}}\int d^d x\frac{1}{g^2}(\frac{1}{4}[D_\alpha, D_\beta][D_\alpha, D_\beta] \\
&-\frac{1}{2}[D_\alpha, \phi_a][D_\alpha, \phi_a] + \frac{1}{4}[\phi_a, \phi_b][\phi_a, \phi_b])_\star,
\end{aligned} \tag{21}$$

and

$$\begin{aligned}
&-\frac{1}{2g^2}\bar{\psi}\Gamma^\mu\left[A_\mu, \psi\right] \\
&\to -\sqrt{\det B}(\frac{1}{2\pi})^{\frac{d}{2}}\int d^d x\frac{1}{2g^2 i}(\bar{\theta}\Gamma_\alpha[D_\alpha, \theta] + \bar{\theta}\Gamma_a[D_a, \theta])_\star.
\end{aligned} \tag{22}$$

where the indices α and β run over the directions parallel to the brane and the indices a and b over the directions transverse to the brane. In the transverse direction, a_a has been replaced by a scalar field ϕ_a. Although we have discussed the momentum space, the coordinate space is also embedded in the matrices of twisted reduced model through the relation $\hat{x}^\mu = C^{\mu\nu}\hat{p}_\nu$. This relation says that the coordinate space is related to the momentum space. This relation is relevant to T-duality[15].

3 Long range interaction with fermionic backgrounds

In this section, we consider quantum corrections of the matrix model. Computing the one-loop effective action between diagonal blocks, the gravitational interactions can be observed and IIB supergravity is expected to be reproduced. Graviton and dilaton exchange are examined in [2, 15]. In [2], one-loop effective action is calculated without fermionic backgrounds. With fermionic backgrounds, gravitino and dilatino exchange processes are expected to be seen. One-loop effective action including fermionic backgrounds is examined in [20] and in [21] in BFSS matrix model. In the bosonic background, leading $1/r^8$ terms in IKKT model[2] and leading $1/r^7$ terms in BFSS model[1] are related by T-duality.

We now derive the one-loop effective action in fermionic backgrounds based on [2, 20]. The matrices A_μ and ψ are divided into the backgrounds and fluctuations:

$$A_\mu = p_\mu + a_\mu, \tag{23}$$

$$\psi = \theta + \varphi. \tag{24}$$

The backgrounds have block-diagonal form:

$$A_\mu^{back} = \begin{pmatrix} p_\mu^{(1)} & & \\ & p_\mu^{(2)} & \\ & & \ddots \end{pmatrix}, \ \ \theta^{back} = \begin{pmatrix} \theta^{(1)} & & \\ & \theta^{(2)} & \\ & & \ddots \end{pmatrix}. \tag{25}$$

p_μ is decomposed into the trace part and traceless part:

$$p_\mu^{(i)} = d_\mu^{(i)}\mathbf{1} + \tilde{p}_\mu^{(i)}, \tag{26}$$

where $d_\mu^{(i)}$ is interpreted as the center of mass coordinates of the i-th blocks. We expand the action (1) up to the second order of the fluctuation and add the following gauge fixing terms to fix the gauge invariance[2],

$$S_{\text{gauge-fix}} = -Tr(\frac{1}{2}[p_\mu, a_\mu]^2 + [p_\mu, b][p_\mu, c]), \tag{27}$$

where c and b are ghosts and anti-ghosts, respectively. The action can be rewritten as

$$\tilde{S} \equiv Tr(\frac{1}{2}a_\mu(P_\lambda^2\delta_{\mu\nu} - 2iF_{\mu\nu})a_\nu - \frac{1}{2}\bar{\varphi}\Gamma^\mu P_\mu\varphi + bP_\lambda^2 c + \bar{\varphi}\Gamma^\mu\Theta a_\mu). \tag{28}$$

where $F_{\mu\nu}$,P_μ and Θ are adjoint operators which act on matrices as follows,

$$\begin{aligned} &P_\mu X = [p_\mu, X], \\ &F_{\mu\nu}X = [f_{\mu\nu}, X] \equiv i[[p_\mu, p_\nu], X], \\ &\Theta X = \theta X - (-)^m X\theta. \end{aligned} \tag{29}$$

where m is 0 for bosonic X and 1 for fermionic X. One loop effective action W is given by the following equation,

$$W = -\log \int da d\varphi db dc e^{-\tilde{S}}. \tag{30}$$

We assume that the blocks are separated far enough from each other. Then the expansion with respect to the inverse powers of the relative distance between the two blocks gives the following expression[20, 21].

$$\begin{aligned}
W &= \sum_{(i,j)} W^{(i,j)}, \\
W^{(i,j)} &= -\frac{1}{2} S\mathcal{T}r^{(i,j)}(F_{\mu\nu}F_{\nu\sigma}F_{\sigma\tau}F_{\tau\mu} - \frac{1}{4}F_{\mu\nu}F_{\mu\nu}F_{\tau\sigma}F_{\tau\sigma})\frac{1}{(d^{(i)}-d^{(j)})^8} \\
&\quad -\frac{1}{2} S\mathcal{T}r^{(i,j)}(\Theta\Gamma^\mu\Gamma^\nu\Gamma^\rho F_{\sigma\mu}F_{\nu\rho}[P_\sigma,\Theta])\frac{1}{(d^{(i)}-d^{(j)})^8} \\
&\quad +W^{(i,j)}_{\theta^4} + O(\frac{1}{(d^{(i)}-d^{(j)})^9}).
\end{aligned} \tag{31}$$

where $S\mathcal{T}r$ means a symmetrized trace in which we average over all possible orderings of the matrices in the trace and treat any commutator as single element. $W^{(i,j)}$ expresses the interaction between the i-th block and j-th block. W_{θ^4} denotes terms including four Θ's. We are not interested in these terms in the present discussions. $(d^{(i)} - d^{(j)})$ is the distance between the center of mass coordinate of the i-th block and that of the j-th block. The terms up to $O(r^{-7})$ cancel each other when backgrounds are restricted to satisfy the matrix model equations of motion (7) and (8). $\mathcal{T}r$ is the trace of the adjoint operators. This expression can be rewritten as the form of the interaction between the diagonal blocks and we take terms which are related to the exchange of fermionic particles [3]:

$$\begin{aligned}
W^{(i,j)} &= 12\frac{1}{(d^{(i)}-d^{(j)})^8} Tr(f^{(i)}_{\mu\lambda}[\bar{\theta}^{(i)}_\alpha, \tilde{p}^{(i)}_\lambda])(\Gamma_\nu)_{\alpha\beta} Tr(f^{(j)}_{\mu\nu}\theta^{(j)}_\beta) \\
&\quad +12\frac{1}{(d^{(i)}-d^{(j)})^8} Tr(f^{(i)}_{\mu\nu}[\bar{\theta}^{(i)}_\alpha, \tilde{p}^{(i)}_\lambda])(\Gamma_\nu)_{\alpha\beta} Tr(f^{(j)}_{\mu\lambda}\theta^{(j)}_\beta) \\
&\quad -6\frac{1}{(d^{(i)}-d^{(j)})^8} Tr([f^{(i)}_{\lambda\rho}, \tilde{p}^{(i)}_\nu]\bar{\theta}^{(i)}_\alpha)(\Gamma_{\lambda\mu\rho})_{\alpha\beta} Tr(f^{(j)}_{\mu\nu}\theta^{(j)}_\beta) \\
&\quad +4\frac{1}{(d^{(i)}-d^{(j)})^8} Tr([\tilde{p}^{(i)}_\lambda, f^{(i)}_{\lambda\rho}]\bar{\theta}^{(i)}_\alpha)(\Gamma_{\rho\nu\mu})_{\alpha\beta} Tr(f^{(j)}_{\mu\nu}\theta^{(j)}_\beta).
\end{aligned} \tag{32}$$

[3]In the paper, we are considering only the interaction which is related to a photon-photon to photino-photino scattering.

Now we can apply these results to noncommutative gauge theory. For simplicity, our discussion is restricted to $U(1)$ noncommutative gauge theory. A graviton exchange process in noncommutative Yang-Mills is investigated in [15]. We consider the following noncommutative backgrounds:

$$A^{back}_\mu = \begin{pmatrix} p_\mu + a^{(1)}_\mu & 0 \\ 0 & p_\mu + a^{(2)}_\mu \end{pmatrix}, \tag{33}$$

$$\psi^{back} = \begin{pmatrix} \theta^{(1)} & 0 \\ 0 & \theta^{(2)} \end{pmatrix}, \tag{34}$$

where p_μ satisfies the noncommutative relation (9). The rank of $B_{\mu\nu}$ is d(=even), that is, the eigenvalues are extended over d dimensional space-time. We set $B_{01} = B_{23} = \cdots \equiv B$ for simplicity. a_μ and θ are considered as photon and photino fields. In the transverse directions, a_a is replaced by scalar fields ϕ_a. By using the mapping rule which is summarized in the previous section, we have obtained the interactions in noncommutative Yang-Mills from (32):

$$\begin{aligned}
W = &-\frac{12}{i}\frac{B^{d-8}}{(2\pi)^d}\int d^dxd^dy(f^{(1)}_{\mu\lambda}(\partial_\lambda\bar{\theta}^{(1)}_\alpha))(x)(\Gamma_\nu)_{\alpha\beta}\frac{1}{(x-y)^8}(f^{(2)}_{\mu\nu}\theta^{(2)}_\beta)(y) \\
&-\frac{12}{i}\frac{B^{d-8}}{(2\pi)^d}\int d^dxd^dy(f^{(1)}_{\mu\nu}(\partial_\lambda\bar{\theta}^{(1)}_\alpha))(x)(\Gamma_\nu)_{\alpha\beta}\frac{1}{(x-y)^8}(f^{(2)}_{\mu\lambda}\theta^{(2)}_\beta)(y) \\
&+\frac{4}{i}\frac{B^{d-8}}{(2\pi)^d}\int d^dxd^dy((\partial_\lambda f^{(1)}_{\lambda\rho})\bar{\theta}^{(1)}_\alpha)(x)(\Gamma_{\rho\nu\mu})_{\alpha\beta}\frac{1}{(x-y)^8}(f^{(2)}_{\mu\nu}\theta^{(2)}_\beta)(y) \\
&+\frac{6}{i}\frac{B^{d-8}}{(2\pi)^d}\int d^dxd^dy((\partial_\nu f^{(1)}_{\lambda\rho})\bar{\theta}^{(1)}_\alpha)(x)(\Gamma_{\lambda\mu\rho})_{\alpha\beta}\frac{1}{(x-y)^8}(f^{(2)}_{\mu\nu}\theta^{(2)}_\beta)(y),
\end{aligned} \tag{35}$$

where $f_{\mu\nu} = -B_{\mu\nu} + \partial_\mu a_\nu - \partial_\nu a_\mu$. We have assumed that the external momenta are small compared to the noncommutative scale. Hence the phase factor which depends on the external momenta is dropped. These terms can be rewritten by using the equations of motion (20) and the following Jacobi identity,

$$\begin{aligned}
&\left(f_{\mu\nu}\partial_\lambda\bar{\theta}\right)(x)G(x-y) + \left(f_{\nu\lambda}\partial_\mu\bar{\theta}\right)(x)G(x-y) + \left(f_{\lambda\mu}\partial_\nu\bar{\theta}\right)(x)G(x-y) \\
&+\left(f_{\mu\nu}\bar{\theta}\right)(x)\partial^x_\lambda G(x-y) + \left(f_{\lambda\mu}\bar{\theta}\right)(x)\partial^x_\nu G(x-y) + \left(f_{\nu\lambda}\bar{\theta}\right)(x)\partial^x_\mu G(x-y) = 0.
\end{aligned} \tag{36}$$

We can show that order $1/r^8$ terms vanish up to terms with four θ's and up to total derivative terms and we obtain the following expression:

$$W = \frac{12}{i}\frac{B^{d-8}}{(2\pi)^d}\int d^dxd^dy(f^{(1)}_{\rho\nu}\bar{\theta}^{(1)}\Gamma_\rho)(x)\partial\!\!\!/^x\frac{1}{(x-y)^8}(\Gamma_\mu f^{(2)}_{\mu\nu}\theta^{(2)})(y). \tag{37}$$

We find that only a order $1/r^9$ term remains because there is a derivative acting on $1/r^8$. The effect of $B_{\mu\nu}$ decouples from the gravitational interaction:

$$\begin{aligned} W &= -8\frac{12}{i}\frac{B^{d-6}}{(2\pi)^d}\int d^dxd^dy(\bar{\theta}^{(1)})(x)\partial\!\!\!/^x\frac{1}{(x-y)^8}(\theta^{(2)})(y) \\ &\quad +\frac{12}{i}\frac{B^{d-8}}{(2\pi)^d}\int d^dxd^dy(\tilde{f}^{(1)}_{\rho\nu}\bar{\theta}^{(1)}\Gamma_\rho)(x)\partial\!\!\!/^x\frac{1}{(x-y)^8}(\Gamma_\mu\tilde{f}^{(2)}_{\mu\nu}\theta^{(2)})(y) \\ &= \frac{12}{i}\frac{B^{d-8}}{(2\pi)^d}\int d^dxd^dy(\tilde{f}^{(1)}_{\rho\nu}\bar{\theta}^{(1)}\Gamma_\rho)(x)\partial\!\!\!/^x\frac{1}{(x-y)^8}(\Gamma_\mu\tilde{f}^{(2)}_{\mu\nu}\theta^{(2)})(y), \end{aligned} \tag{38}$$

where $\tilde{f}_{\mu\nu}=\partial_\mu a_\nu-\partial_\nu a_\mu$. In this way, we have obtained the long range interactions which decay as $1/r^9$ in noncommutative Yang-Mills. This behavior is independent of the dimensionality of the backgrounds. This long range interaction is interpreted as the propagation of the massless fermions in ten dimensional supergravity and arises from the nonplanar diagram in noncommutative Yang-Mills[14]. In the next section, we see that this interaction is interpreted in terms of the gravitino exchange in ten dimensional supergravity.

Before finishing this section, let us consider a supersymmetry current which is a Noether current associated with supersymmetry. IIB matrix model has $\mathcal{N}=2$ supersymmetry (2) and (3). We can determine two supersymmetry currents in the matrix model by the Noether method:

$$\begin{aligned} \hat{S}^{\mu(1)} &= \frac{1}{2g^2}\Gamma^\mu\psi \\ \hat{S}^{\mu(2)} &= \frac{1}{2g^2}f_{\sigma\rho}\Gamma^{\sigma\rho}\Gamma^\mu\psi=\frac{i}{2g^2}[A_\sigma,A_\rho]\Gamma^{\sigma\rho}\Gamma^\mu\psi. \end{aligned} \tag{39}$$

The first supersymmetry current is associated with (2) and the second is associated with (3). The supersymmetry algebra are constructed in [22, 23]. The corresponding currents in noncommutative Yang-Mills theory are given by

$$\begin{aligned} S^{\mu(1)}(x) &= \frac{1}{(2\pi)^{\frac{d}{2}}}\frac{1}{2g^2}B^{\frac{d}{2}-3}\Gamma^\mu\theta(x) \\ S^{\mu(2)}(x) &= \frac{1}{(2\pi)^{\frac{d}{2}}}\frac{1}{4g^2}B^{\frac{d}{2}-4}f_{\sigma\rho}\star(\Gamma^{\sigma\rho}\Gamma^\mu\theta)(x), \end{aligned} \tag{40}$$

where $f_{\sigma\rho}=\partial_\sigma a_\rho-\partial_\rho a_\sigma+[a_\sigma,a_\rho]_\star$. $B_{\mu\nu}$ part, which comes from (18), in $\hat{S}^{\mu(2)}$ is proportional to $\hat{S}^{\mu(1)}$.

These currents are shown to be conserved by using the equation of motion and the Bianchi identity,

$$[D_\mu, S^\mu(x)]_\star = 0 \tag{41}$$

In the next section, we investigate the interaction between $S_\mu^{(2)}$ supercurrents via exchange of a gravitino in ten dimensional supergravity. We find that supergravity gives the same expression as (38). We comment about $S_\mu^{(1)}$ supercurrent in section 5.

4 Gravitino exchange in supergravity

In this section, we consider a photon-photon to photino-photino scattering via exchange of a gravitino, which is regarded as the supercurrent interactions in ten dimensional supergravity.

The interaction which is obtained in the matrix model calculation is shown to be regarded as the interaction between $S_\mu^{(2)}$ supersymmetry currents in supergravity. We also interprete this interaction as photon-photon to photino-photino scattering via exchange of a gravitino or as the non-planar one-loop diagrams in the two-point function of supercurrents in noncommutative Yang-Mills.

We consider the following gravitino action in ten dimensional supergravity,

$$S_{\text{kinetic}} = \frac{i}{2}\int d^{10}x \bar{\psi}_\mu(x)\Gamma^{\mu\rho\nu}\partial_\rho\psi_\nu(x), \tag{42}$$

and determine the gravitino propagator. Supergravity theory has local supersymmetry and therefore a gauge fixing term must be added to obtain the propagator[26]. We choose the following gauge fixing term,

$$S_{\text{gauge-fix}} = -i\int d^{10}x \bar{\psi}_\mu(x)\Gamma^\mu\Gamma^\rho\Gamma^\nu\partial_\rho\psi_\nu(x). \tag{43}$$

This gauge is analogous to Feynman gauge in QED. Other gauges (ex. Landau gauge) have three derivatives. Such a term does not appear in the matrix model calculation, therefore Feynman gauge is adequate for comparing supergravity and the matrix model calculation. The propagator is determined by the following equation,

$$i(-\frac{1}{2}\Gamma^{\mu\nu\rho}\partial_\rho^x - \frac{1}{i}\Gamma^\mu\Gamma^\rho\Gamma^\nu\partial_\rho^x)\langle\psi_\nu(x)\bar{\psi}_\tau(y)\rangle = \eta_\tau^\mu\delta(x-y), \tag{44}$$

and we find

$$\langle\psi_\nu(x)\bar{\psi}_\tau(y)\rangle = \int \frac{d^{10}k}{i(2\pi)^{10}}\frac{1}{4k^2}(\Gamma_\tau i\not{k}\Gamma_\nu - 6\eta_{\nu\tau}i\not{k})e^{ik\cdot(x-y)}$$

$$= \frac{3}{i8\pi^5}(\Gamma_\tau \partial\!\!\!/ \Gamma_\nu - 6\eta_{\nu\tau}\partial\!\!\!/)\frac{1}{(x-y)^8}. \tag{45}$$

The gravitino field is assumed to have a linear coupling with the supercurrent by the form

$$S = \kappa \int d^d x (\bar{\psi}_\mu(x) S^{\mu(2)}(x) + \bar{S}^{\mu(2)}(x)\psi_\mu(x)), \tag{46}$$

where we denote a photon-photino-gravitino coupling constant as κ.

A tree level gravitino exchange diagram can be calculated by

$$V = \kappa^2 \int d^d x d^d y \bar{S}^{\mu(2)}(x)\langle\psi_\mu(x)\bar{\psi}_\nu(y)\rangle S^{\nu(2)}(y). \tag{47}$$

We assume that the fields which appear in the external lines satisfy the on-shell conditions:

$$\Gamma_\mu \partial_\mu \theta = 0, \tag{48}$$

$$\partial_\mu f_{\mu\nu} = \bar{\theta}\Gamma_\nu \theta. \tag{49}$$

We denote a normalization factor of $S^{\mu(2)}$ as C. Then we have

$$\begin{aligned}
V &= -C^2\kappa^2 \frac{3}{i8\pi^5}\int d^d x d^d y (f_{\rho\sigma}\bar{\theta}\Gamma^\mu\Gamma^{\sigma\rho})(x)(\Gamma_\nu \partial\!\!\!/ \Gamma_\mu - 6\eta_{\mu\nu}\partial\!\!\!/)\frac{1}{(x-y)^8}(\Gamma^{\tau\lambda}\Gamma^\nu f_{\tau\lambda}\theta)(y) \\
&= C^2\kappa^2\frac{9}{i2\pi^5}\int d^d x d^d y (f_{\rho\sigma}\bar{\theta}\Gamma_{\sigma\rho})(x)\partial\!\!\!/ \frac{1}{(x-y)^8}(\Gamma_{\tau\lambda}f_{\tau\lambda}\theta)(y) \\
&\quad -C^2\kappa^2\frac{12}{i\pi^5}\int d^d x d^d y (f_{\rho\sigma}\bar{\theta}\Gamma_{\sigma\rho})(x)\partial_\tau \frac{1}{(x-y)^8}(\Gamma_\lambda f_{\tau\lambda}\theta)(y) \\
&\quad +C^2\kappa^2\frac{48}{i\pi^5}\int d^d x d^d y (f_{\rho\sigma}\bar{\theta}\Gamma_\sigma)(x)\partial\!\!\!/ \frac{1}{(x-y)^8}(\Gamma_\lambda f_{\rho\lambda}\theta)(y) \\
&= C^2\kappa^2\frac{3}{i2\pi^5}\int d^d x d^d y (f_{\rho\sigma}\bar{\theta}\Gamma_{\sigma\rho})(x)\partial\!\!\!/ \frac{1}{(x-y)^8}(\Gamma_{\tau\lambda}f_{\tau\lambda}\theta)(y) \\
&\quad +C^2\kappa^2\frac{48}{i\pi^5}\int d^d x d^d y (f_{\rho\sigma}\bar{\theta}\Gamma_\sigma)(x)\partial\!\!\!/ \frac{1}{(x-y)^8}(\Gamma_\lambda f_{\rho\lambda}\theta)(y).
\end{aligned} \tag{50}$$

In the last equality, we have used the equations of motion and ignore the total derivative terms. In view of the tensor structure, the first term is identical with the dilatino exchange while the second term is specific to the gravitino exchange.

By the same procedure, we next consider a spin-1/2 component of the supercurrent:

$$S^{(2)}(x) \equiv \beta\Gamma_\mu S^{\mu(2)}(x) = \beta 6C f_{\rho\sigma}\Gamma^{\rho\sigma}\theta(x), \tag{51}$$

where we denote β as a normalization factor. It is expected that this component couples to a dilatino field. The dilatino kinetic term has the following form:

$$S = \frac{i}{2} \int d^{10}x \bar{\lambda}(x) \Gamma^\mu \partial_\mu \lambda(x). \tag{52}$$

Dilatino propagator is given by

$$\begin{aligned} \langle \lambda(x) \bar{\lambda}(y) \rangle &= -\int \frac{d^{10}k}{i(2\pi)^{10}} \frac{2i\not{k}}{k^2} e^{ik\cdot(x-y)} \\ &= -\frac{3}{i\pi^5} \not{\partial} \frac{1}{(x-y)^8}. \end{aligned} \tag{53}$$

A dilatino field has a coupling with the spin-1/2 component of the supercurrent:

$$S = \kappa \int d^d x (\bar{\lambda}(x) S^{(2)}(x) + \bar{S}^{(2)}(x) \lambda(x)). \tag{54}$$

Therefore interaction between spin-1/2 component is calculated as follows

$$\begin{aligned} V &= \kappa^2 \int d^d x d^d y \bar{S}^{(2)}(x) \langle \lambda(x) \bar{\lambda}(y) \rangle S^{(2)}(y) \\ &= -\beta^2 C^2 \kappa^2 \frac{108}{i\pi^5} \int d^d x d^d y (f_{\rho\sigma} \bar{\theta} \Gamma^{\sigma\rho})(x) \not{\partial} \frac{1}{(x-y)^8} (\Gamma^{\tau\lambda} f_{\tau\lambda} \theta)(y). \end{aligned} \tag{55}$$

From (50) and (55), we have

$$\begin{aligned} V = C^2 \kappa^2 \frac{1}{i\pi^5} (\frac{3}{2} - 108\beta^2) \int d^d x d^d y (f_{\rho\sigma} \bar{\theta} \Gamma_{\sigma\rho})(x) \not{\partial} \frac{1}{(x-y)^8} (\Gamma_{\tau\lambda} f_{\tau\lambda} \theta)(y) \\ + C^2 \kappa^2 \frac{48}{i\pi^5} \int d^d x d^d y (f_{\rho\sigma} \bar{\theta} \Gamma_\sigma)(x) \not{\partial} \frac{1}{(x-y)^8} (\Gamma_\lambda f_{\rho\lambda} \theta)(y). \end{aligned} \tag{56}$$

We compare this supergravity result to the matrix model result(38). It turns out that κ is determined as follows,

$$\begin{aligned} \kappa^2 &= 4\pi^5 g^4 \\ &= 16\pi^7 g_s^2 \alpha'^4. \end{aligned} \tag{57}$$

In the last equality, we have expressed g^2 by string coupling g_s and Regge slope α' according to the relation $g^2 = 2\pi g_s \alpha'^2$ [2, 15]. We also find that β is given by

$$\beta = \frac{1}{6\sqrt{2}}. \tag{58}$$

5 Conclusions and discussions

In this paper, we have studied the nonplanar two point function of the supercurrents in noncommutative Yang-Mills. In nonplanar diagrams, infrared divergence appears after integrating high momentum modes. This is a particular phenomenon in noncommutative theories and is interpreted to arise from the propagation of massless particles. We have analyzed this interaction from the block-block interaction in the matrix model using the connection between the matrix model and noncommutative Yang-Mills theory. Block-block interactions in IIB matrix model are well investigated. We examined the block-block interactions with fermionic backgrounds at order $1/r^8$ and mapped to noncommutative Yang-Mills. Then we have obtained the interactions which decay as $1/r^9$ in noncommutative Yang-Mills, which is interpreted as the propagation of fermionic particle in IIB supergravity. Comparing the matrix model result to supergravity result, we observed the gravitino and dilatino exchange processes in the matrix model. We also find that one of the two gravitinos in IIB supergravity couples to a $S^{(2)}_\mu$ supersymmetry current.

We have mapped the long range interactions at order $1/r^8$ in the matrix model to noncommutative Yang-Mills. However the long range interaction at order $1/r^9$ in noncommutative Yang-Mills can appear not only order at $1/r^8$ in the matrix model calculation, which is considered in this paper, but also at $1/r^9$. Therefore we have to pay attention to sub-leading terms (order $1/r^9$ terms) in the matrix model calculation. These terms are calculated in [21] in BFSS matrix model. One loop amplitudes of BFSS matrix model and those of IIB matrix model are related each other by T-duality. We can expect that similar terms appear in IIB matrix model calculation. According to [21], there appear order $1/r^9$ terms which are proportional to the insertion of $d_\mu P^\mu$ into leading terms in (31). Other terms at order $1/r^9$ contain three $F_{\mu\nu}$'s. Therefore we only need to consider order $1/r^8$ terms in the matrix model interactions for our investigations of gravitino exchange processes.

We now comment about another gravitino. IIB matrix model has $\mathcal{N} = 2$ supersymmetry. We have two supercurrents (39) in the matrix model. However after mapping to noncommutative Yang-Mills, half of the supersymmetry is spontaneously broken due to the backgrounds. Therefore $S^{(1)}_\mu$ supercurrent is not a supersymmetry generator after mapping to noncommutative Yang-Mills. We easily find that $S^{(1)}_\mu$ current does not couple to gravity because $B_{\mu\nu}$ part decouples from the gravitational interaction as in (38). To find another gravitino is a problem which is considered in a separate paper.

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References

[1] T.Banks, W.Fischler, S.Shenker and L.Susskind, *M theory as a matrix model: a conjecture*, Phys.Rev.D55(1997)5112, hep-th/9610043.

[2] N.Ishibashi, H.Kawai, Y.Kitazawa and A.Tsuchiya, *A Large N reduced model as superstring*, Nucl.Phys.B498(1997)467, hep-th/9612115.

[3] J.Polchinski, *TASI Lectures on D-Branes*, hep-th/9611050.

[4] W.Taylor, *Lectures on D-branes, Gauge Theory and M(atrices)*, hep-th/9801182.

[5] T.Eguchi and H.Kawai, Phys.Rev.Lett.48(1982)1063.

G.Parisi, Phys.Lett.112B(1982)463.

D.Gross and Y.Kitazawa, Nucl.Phys.B206(1982)440.

G.Bhanot, U.Heller and H.Neuberger, Phys.Lett113B(1982)47.

S.Das and S.Wadia, Phys.Lett.117B(1982)228.

[6] A.Gonzales-Arroya and M.Okawa, Phys.Rev.D27(1983)2397.

[7] A Connes, M. R. Douglas and A. Schwarz, *Noncommutative Geometry and Matrix Theory : Compactification on Tori*, JHEP9802(1998)003, hep-th/9711162.

[8] N.Seiberg and E.Witten, *String theory and Noncommutative Geometry*,

JHEP9909(1999)032, hep-th/9908056.

[9] T.Filk, *Divergence in a field theory on quantum space*, Phys.Lett.B376(1996)53.

[10] D.Bigatti and L.Susskind, *Magnetic fields, branes and noncommutative geometry* Phys.Rev.D62(2000)066004, hep-th/9908056.

[11] H.Aoki, S.Iso, H.Kawai, Y.Kitazawa, T.Tada and A.Tsuchiya, *IIB Matrix Model*,

Prog.Theor.Phys.Suppl.134(1999)47, hep-th/9908038.

[12] H.Aoki, N.Ishibashi, S.Iso, H.Kawai, Y.Kitazawa and T.Tada, *Noncommutative Yang-Mills in IIB Matrix Model*, Nucl.Phys.B565(2000)176, hep-th/9908141.

[13] N.Ishibashi, S.Iso, H.Kawai and Y.Kitazawa, *Wilson loops in Noncommutative Yang-Mills* , Nucl.Phys.B573(2000)573, hep-th/9910004.

[14] S.Iso, H.Kawai and Y.Kitazawa, *Bi-local Fields in Noncommutative Field Theory*,

Nucl.Phys.B576(2000)375, hep-th/0001027.

[15] N.Ishibashi, S.Iso, H.Kawai and Y.Kitazawa, *String Scale in Noncommutative Yang-Mills*, Nucl.Phys.B583(2000)159, hep-th/0004038.

[16] Y.Kimura and Y.Kitazawa, *IIB Matrix Model with D1-D5 Backgrounds*, Nucl.Phys.B581(2000)295, hep-th/9912258.

[17] I.Bars, D.Minic, *Non-Commutative Geometry on a Discrete Periodic Lattice and Gauge Theory*, hep-th/9910091.

[18] J.Ambjorn, Y.M.Makeenko, J.Nishimura and R.J.Szabo, *Finite N Matrix Models of Noncommutative Gauge Theory*, JHEP9911(1999)029, hep-th/9911041.

[19] J.Ambjorn, Y.M.Makeenko, J.Nishimura and R.J.Szabo, *Lattice Gauge Fields and Discrete Noncommutative Yang-Mills Theory*, JHEP0005(2000)023, hep-th/0004147.

[20] T.Suyama and A.Tsuchiya, talk presented at JPS meeting 1999 and private communication.

[21] W.Taylor and M.V.Raamsdonk, *Supergravity currents and linearized interactions for Matrix Theory configurations with fermionic backgrounds*, JHEP9904(1999)013, hep-th/9812239.

[22] T.Banks, N.Seiberg and S.Shenker, *Branes from Matrices*, Nucl.Phys.B490(1997)91, hep-th/9612157.

[23] I.Chepelev, Y.Makeenko and K.Zarembo, *Properties of D-Branes in Matrix Model of IIB Superstring*, Phys.Lett.B400(1997)43, hep-th/9701151.

[24] S.Minwalla, M.V.Raamsdonk and N.Seiberg, *Noncommutative Perturbative Dynamics*, hep-th/9912072.

M.V.Raamsdonk and N.Seiberg, *Comments on Noncommutative Perturbative Dynamics*, JHEP0003(2000)035, hep-th/0002186.

[25] M.Hayakawa, *Perturbative analysis on infrared aspects of noncommutative QED on* R^4, Phys.Lett.B478(2000)394, hep-th/9912094:

Perturbative analysis on infrared and ultraviolet aspects of noncommutative QED on R^4, hep-th/9912167.

[26] P.V.Nieuwenhuizen, Phys.Rep.68(1981)189.